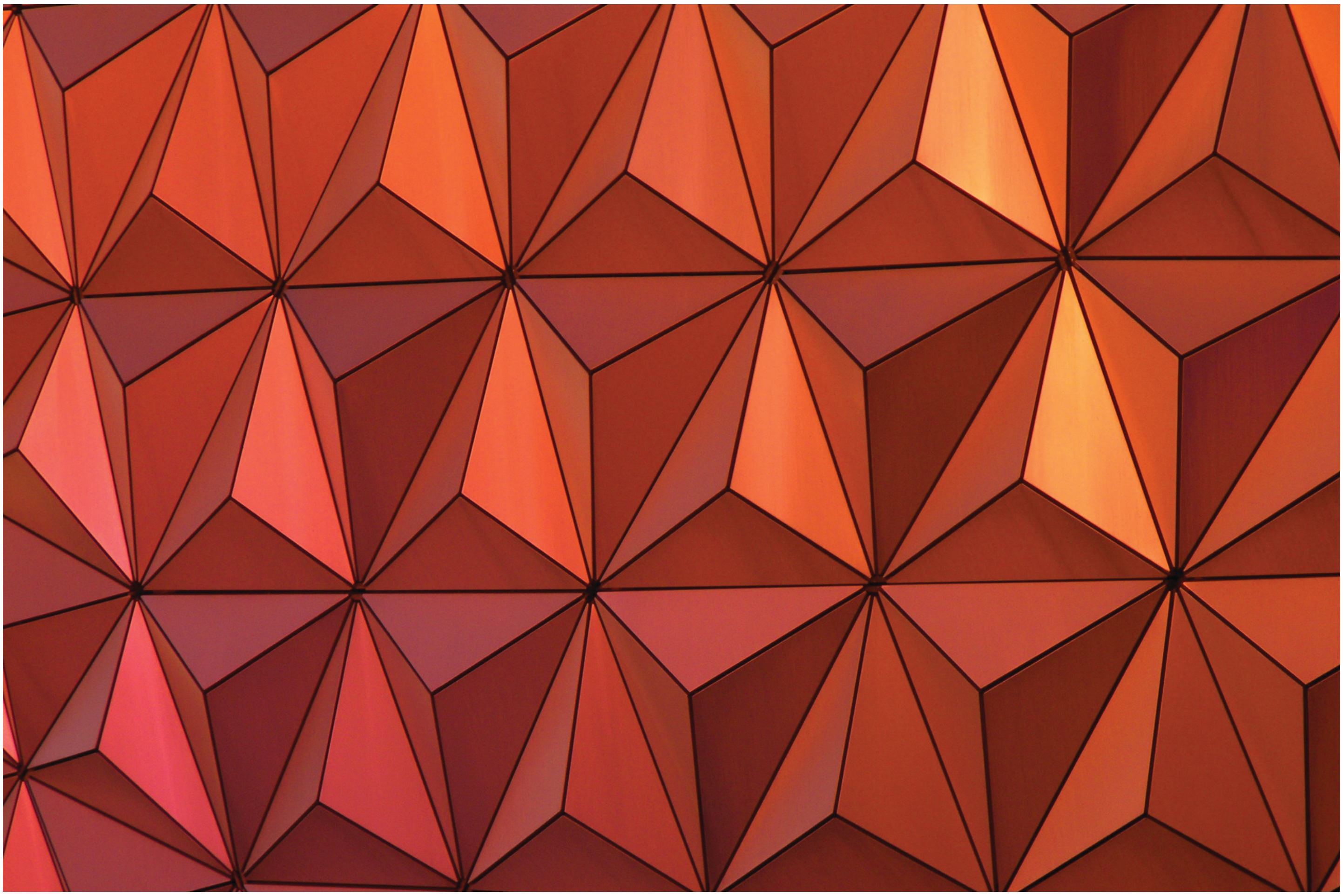
Geometry



About this free course

This free course provides a sample of level 1 study in Mathematics:
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Contents

Introduction

This free course looks at various aspects of shape and space. It uses a lot of mathematical vocabulary, so you should make sure that you are clear about the precise meaning of words such as circumference, parallel, similar and cross-section. You may find it helpful to note down the meaning of each new word, perhaps illustrating it with a diagram.

This OpenLearn course provides a sample of level 1 study in Mathematics.

Learning Outcomes

After studying this course, you should be able to:

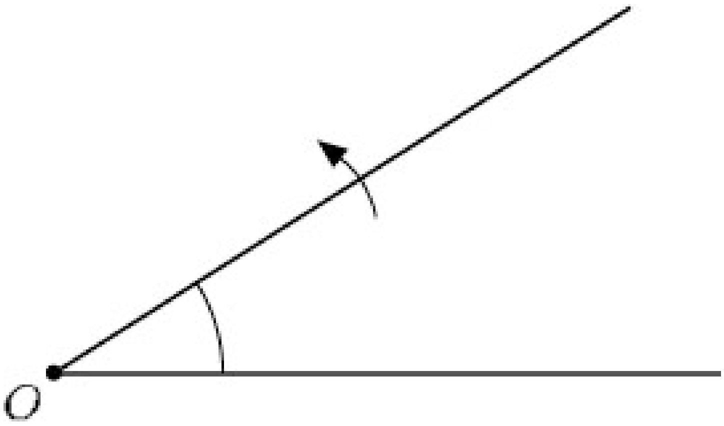
- understand geometrical terminology for angles, triangles, quadrilaterals and circles
- measure angles using a protractor
- use geometrical results to determine unknown angles
- recognise line and rotational symmetries
- find the areas of triangles, quadrilaterals and circles and shapes based on these.

1 Angles

1.1 Angles, notation and measurement

In everyday language, the word 'angle' is often used to mean the space between two lines ('The two roads met at a sharp angle') or a rotation ('Turn the wheel through a large angle'). Both of these senses are used in mathematics, but it is probably easier to start by thinking of an angle in terms of the second of these – as a rotation.

The diagram below shows a fixed arm and a rotating arm (with the arrow), which are joined together at *O*, forming an angle between them. Imagine that the rotating arm, which is pivoted at *O*, initially rests on top of the fixed arm and that it then rotates in the direction of the arrow. Focus on the size of the marked angle between the arms.



At first the angle is quite sharp, but it becomes less so. It then becomes a right angle, and subsequently gets much blunter until the two arms form a straight line. Then it starts to turn back upon itself, passing through a three-quarter turn and, when the rotating arm gets back to the start, it rests on top of the fixed arm again.

The most common unit for expressing angles is degrees, denoted by °, with a complete turn or revolution being equal to 360°. Angles can also be measured in *radians*, and you will meet this unit of measure if you study further maths, science or technology courses.

Acute angle

Any angle that is less than a quarter turn; that is, less than 90°. An example of an acute angle is the angle that a door makes with a doorframe when it is ajar.

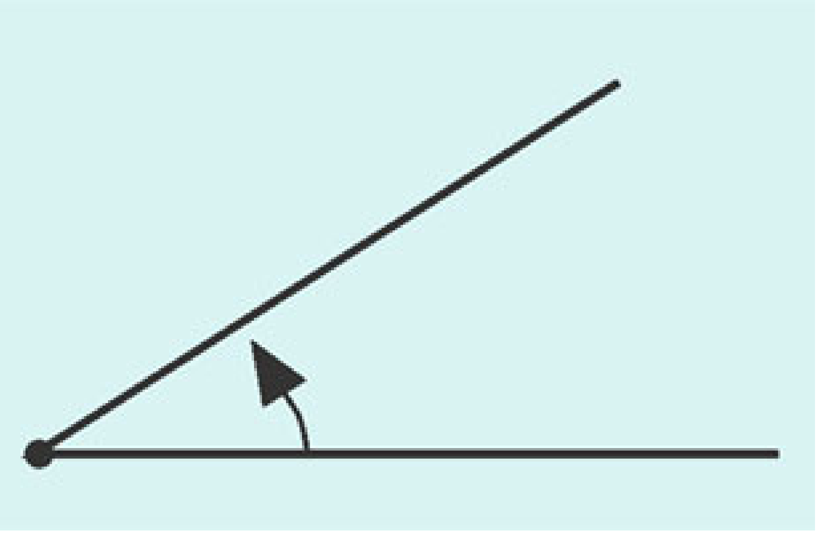


Figure ang1

Right angle

The angle that corresponds to a quarter turn; it is exactly 90°. The angles at the corners of most doors, books and windows are right angles.

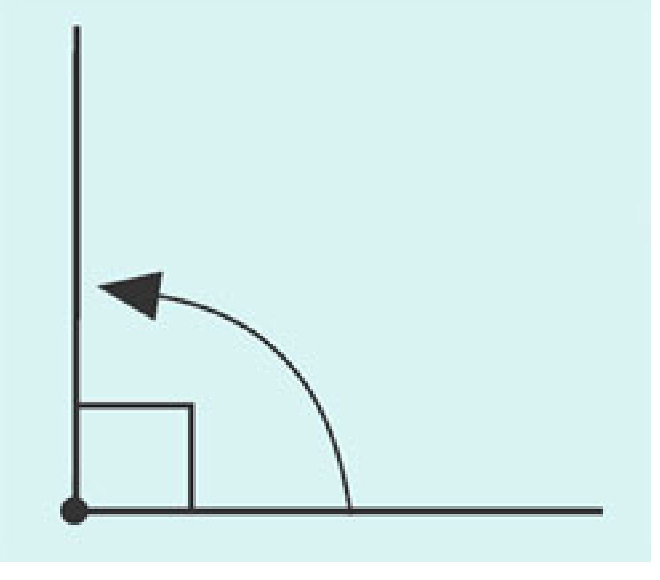


Figure ang2

Obtuse angle

Any angle that is between a quarter turn and a half turn; that is, between 90° and 180°. An example is the angle between the blades of a pair of scissors when they are open as wide as possible.

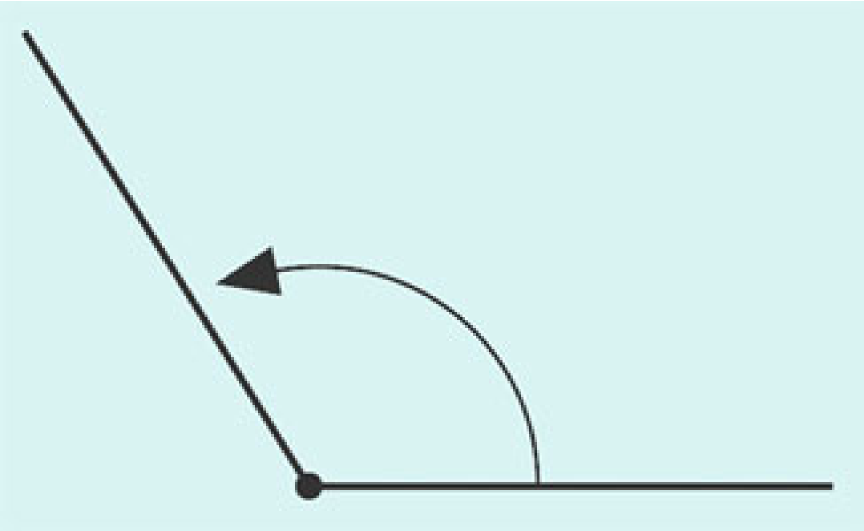
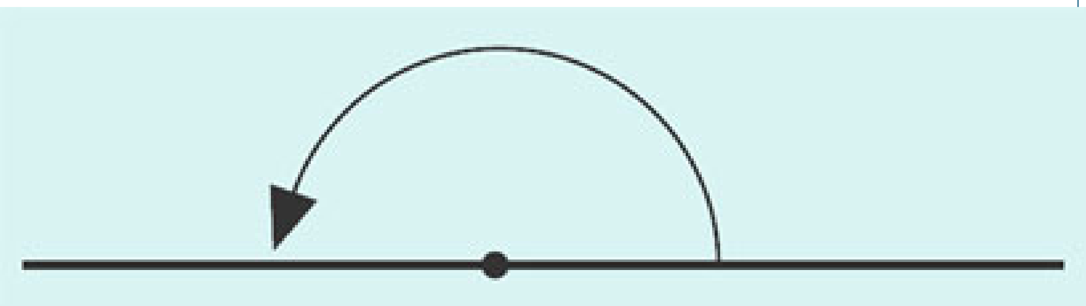


Figure ang3

Half turn (Straight angle)

This corresponds to a straight line; it is exactly 180°. The pages of an open book that is lying flat approximately describe a half turn.



Reflex angle

Any angle that is between a half turn and a complete turn; that is, between 180° and 360°. When a box is opened and the hinged lid falls back so as to rest on the surface on which the box is standing, the angle that the lid turns through is a reflex angle.

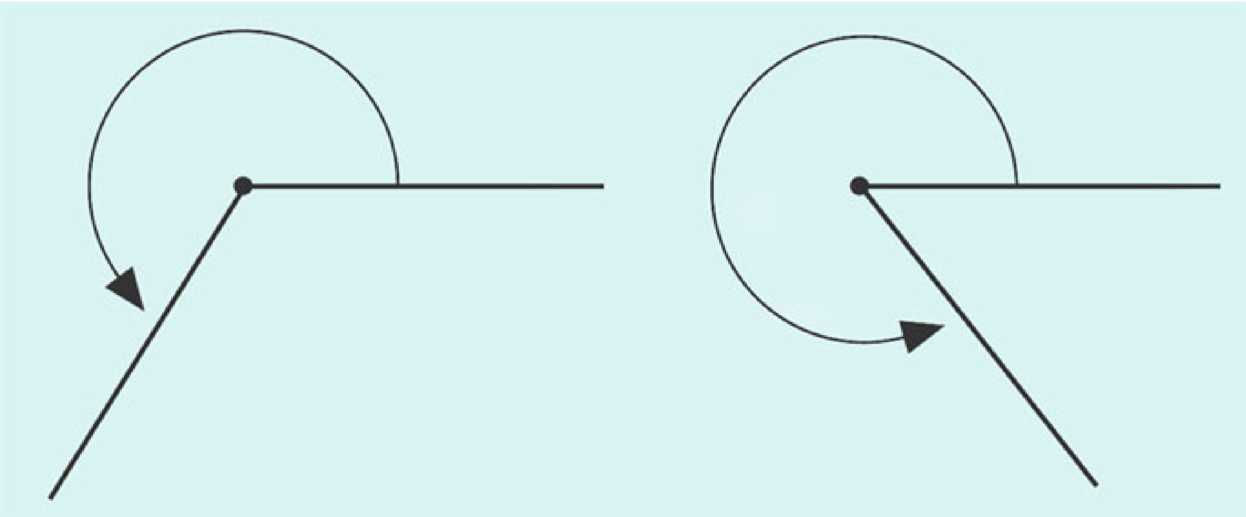


Figure ang5

Complete turn

This corresponds to a complete turn, or one revolution; it is exactly 360°. This is the angle that the minute hand of a clock turns through in an hour.

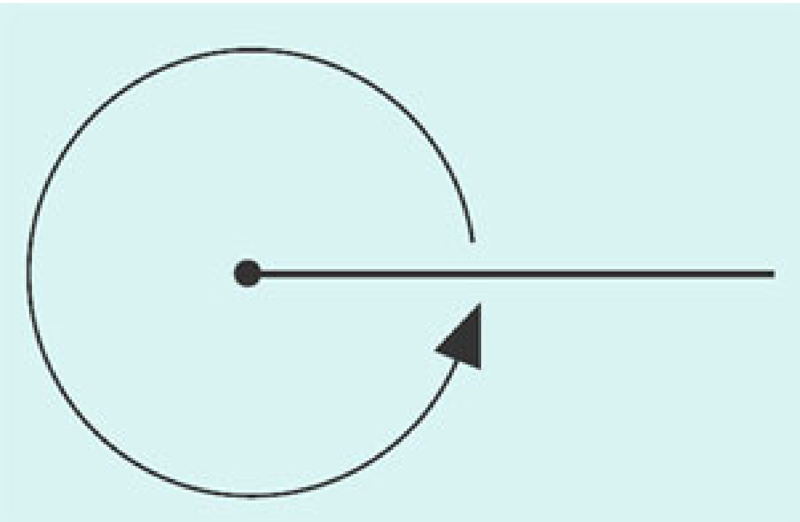
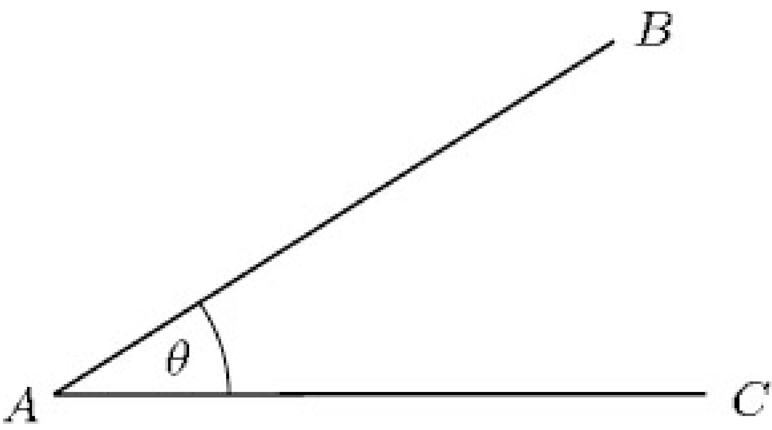


Figure ang6

Remember that if the angle between two straight lines is 90°, then the lines are said to be **perpendicular** to each other.

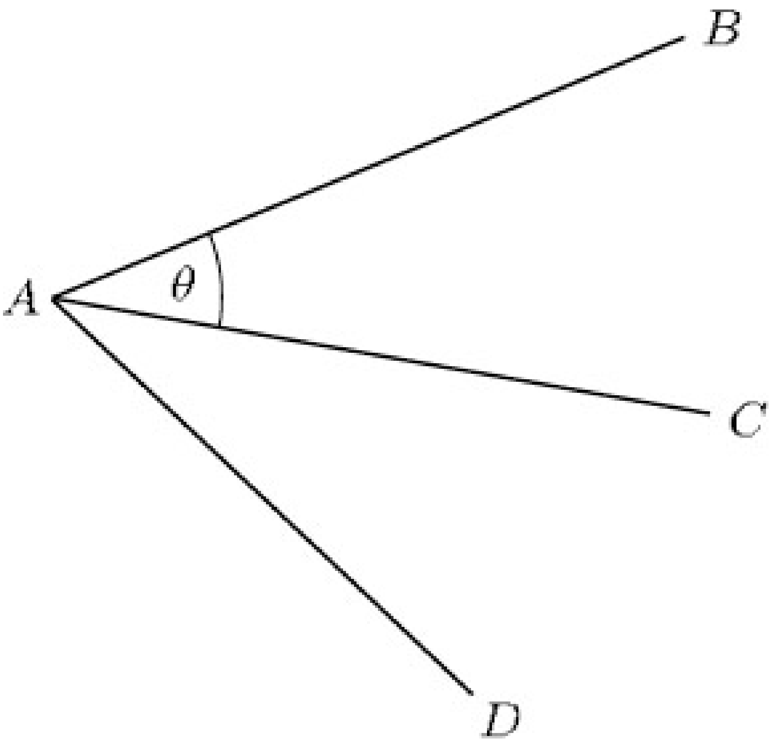
Sometimes it is necessary to refer to a turn that is more than one complete revolution, and so is greater than 360°. An example is the angle that the minute hand of a clock turns through in a period of 12 hours: each complete revolution of the minute hand amounts to 360°, so twelve revolutions amount to 12 × 360° = 4320°.

Several different notations are used for labelling angles. For example, the angle below can be referred to as 'angle BAC' and written as $B\hat{A}C$ or $\angle BAC$, or it can be referred to as the angle 'theta' and labelled θ.



Alternatively, an angle may be denoted by the label on the vertex but with a hat on it. The vertex is another name for the 'corner' of an angle. For instance, the angle θ above may be denoted by $\hat{A}$, which is read as 'angle *A*'.

This notation can be ambiguous if there is more than one angle at the vertex, as in the example below.



In such cases, θ can be specified as $C\hat{A}B$, $B\hat{A}C$, $\angle CAB$ or $\angle BAC$ – the middle letter indicates the vertex and the two outer letters identify the 'arms' of the angle.

Try some yourself

Question 1

What angles do the hour hand and the minute hand of a clock turn through in five hours?

Answer

Every hour the minute hand turns through 360°. It will have made five such revolutions in five hours. This amounts to 1800°.

The hour hand turns through 30° every hour ($\frac{1}{12}$ of 360°). In five hours it will turn through 5 × 30° = 150°.

Question 2

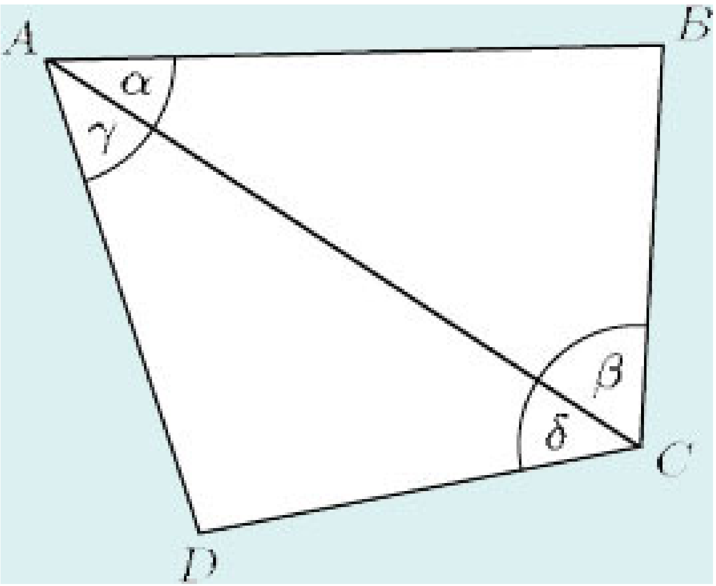
Give an alternative notation for labelling each of these angles in the diagram below.

(a) α

(b) β

(c) $D\hat{A}C$

(d) $\angle ACD$



Answer

(a) $C\hat{A}B$ or $\angle CAB$.

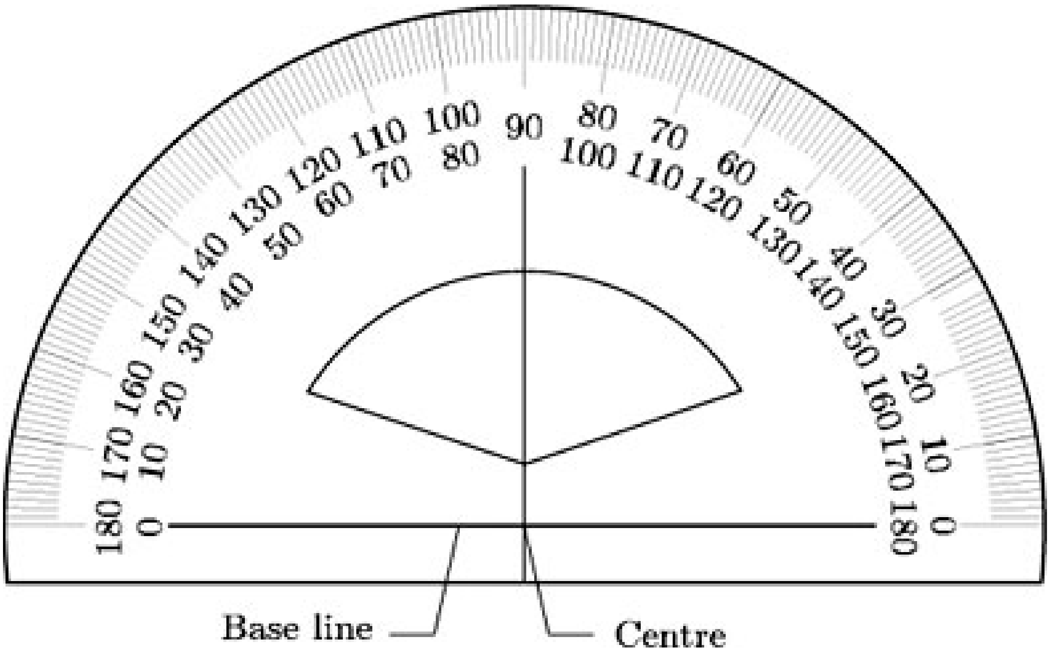
(b) $B\hat{C}A$ or $\angle BCA$.

(c) γ or $\angle DAC$.

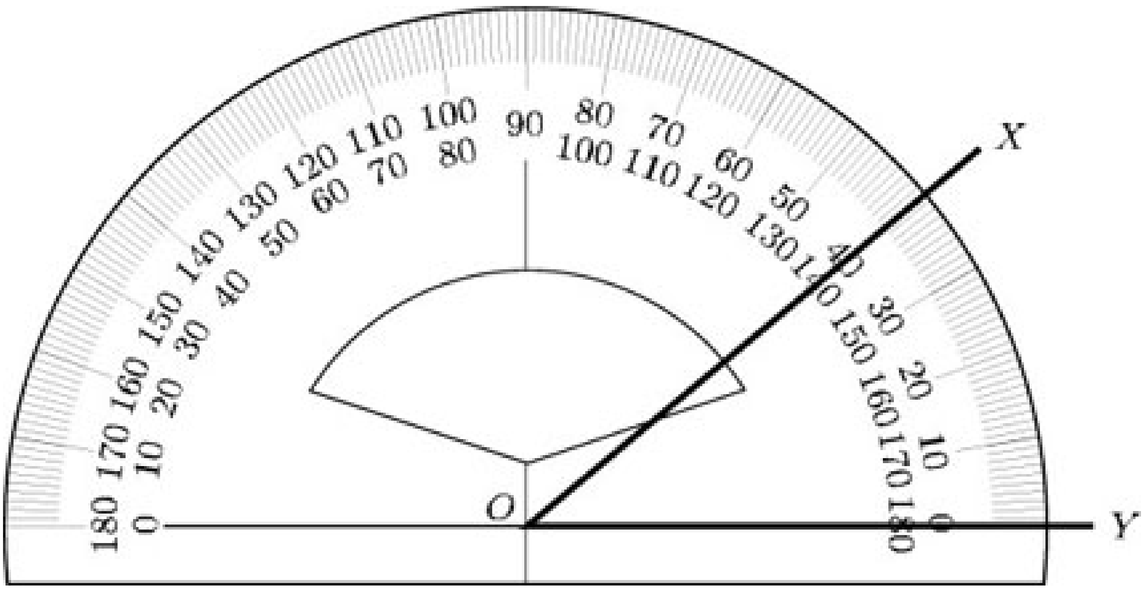
(d) δ or $A\hat{C}D$.

1.2 How to measure an angle

To measure an angle you need a protractor. The protractor shown here is a semicircle that is graduated to measure angles from 0° to 180°. It is also possible to buy circular protractors that measure angles from 0° to 360°.

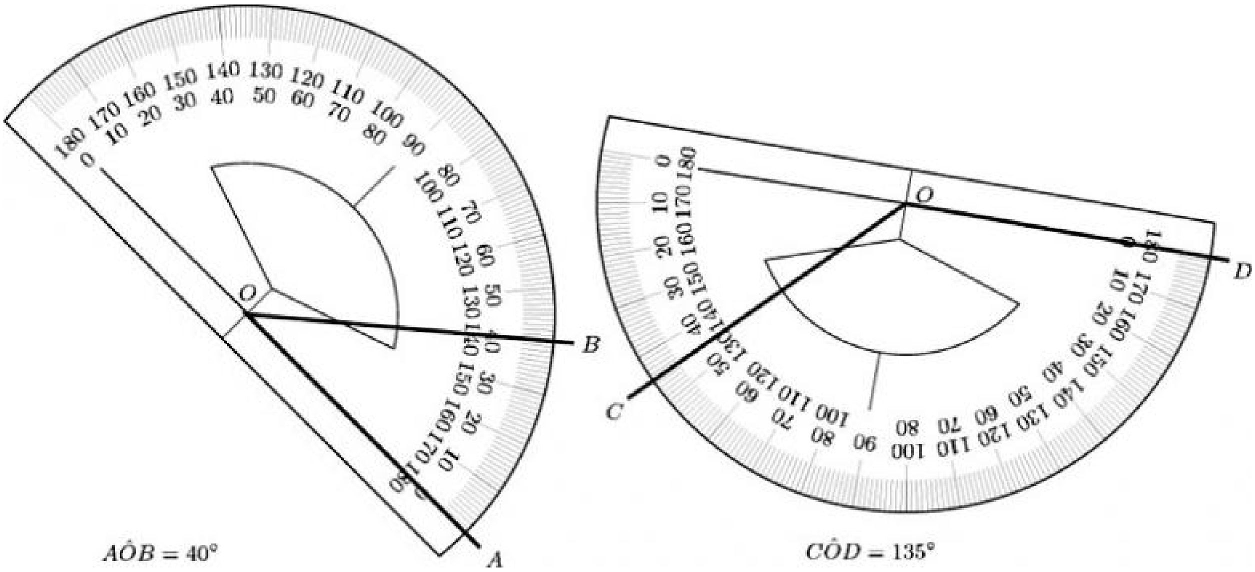


The diagram below indicates how the protractor should be positioned in order to measure an angle. Place the base line of the protractor on one arm of the angle, with the centre *O* on the vertex. The angle can then be read straight from the scale. Here $\angle Y\hat{O}X$ = 40° (*not* 140°).



Be careful to use the correct scale. In this case the angle extends from the line *OY* up to the line *OX*, so use the scale that shows *OY* as 0° – the outer scale in this instance.

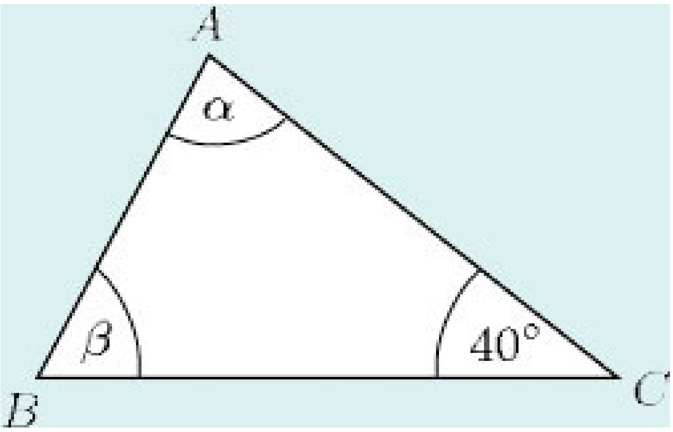
In the above example, one of the arms of the angle is horizontal. However, sometimes you may find that you need to position the protractor in an awkward position in order to measure an angle.



You can also use a protractor to construct an angle accurately, but once you have drawn the angle, be on the safe side and measure it to check that it is correct.

Try some yourself

Question 1



(a) How would you refer to angle α in this triangle by means of the letters *A*, *B* and *C*?

(b) Measure *α* with a protractor , if you have one, or otherwise estimate it.

(c) What type of angle is *α*?

(d) Find *β* without using a protractor using the fact that the three angles of a triangle always add up to 180°.

Answer

(a) Any of the following could be used: $\hat{A}$, $B\hat{A}C$, $\angle BAC$, $C\hat{A}B$, $\angle CAB$.

(b) $\alpha = 80°$.

(c) Because *α* is less than 90°, it is an acute angle.

(d) As the three angles of a triangle always add up to 180°,

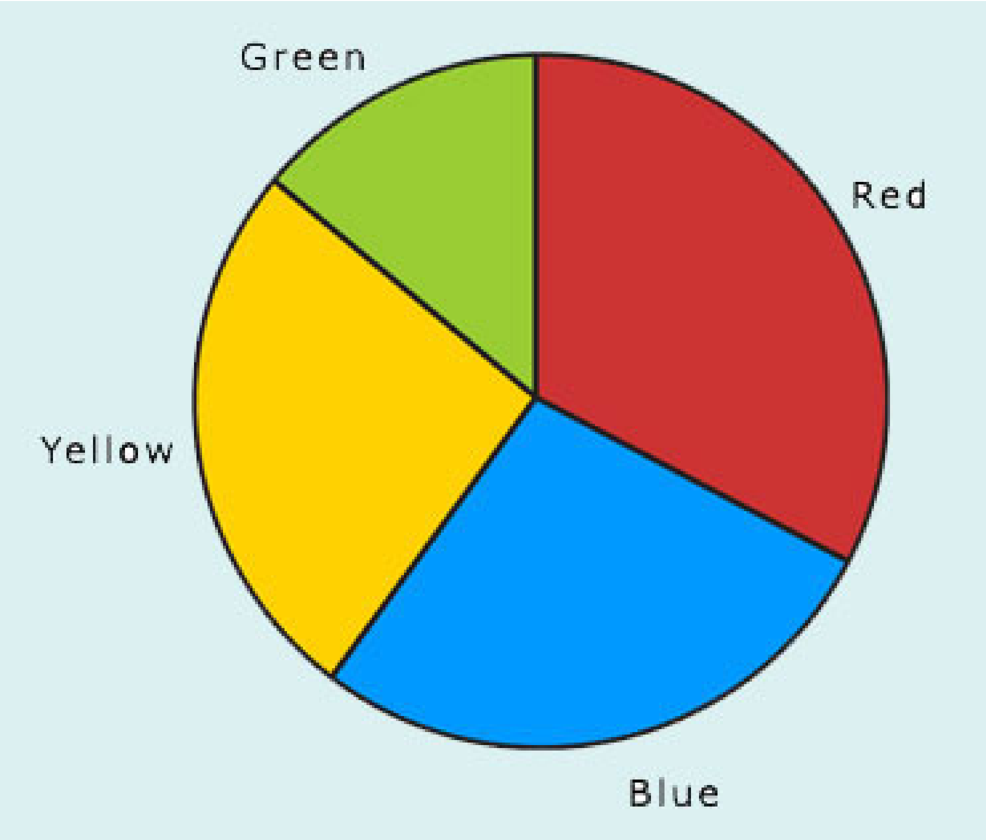
$$\beta + 80° + 40° = 180°.$$

Therefore

$$\beta = 180° - 120° = 60°.$$

Question 2

2 This pie chart shows the proportions of people voting for four parties in a local election.



(a) Measure the angles of the four slices of the pie with your protractor or estimate them if you don't have a protractor.

(b) Check your measurements by ensuring that the angles add up to 360°.

(c) Work out the percentage of the total vote polled by each of the four parties.

Answer

(a) Red party: 120°.

Blue party: 95°.

Yellow party: 95°.

Green party: 50°.

(b) 120° + 95° + 95° + 50° = 360°.

(c) Since 360° represents 100%, 1° will represent $\frac{1}{360} \times 100\%$ or $\frac{100}{360}\%$.

So the Red party polled