PROBLEM BASED LEARNING IN BASIC PHYSICS - III

A.K.Mody
V.E.S. College of Arts, Science and Commerce
Sindhi Society, Chembur, Mumbai

H.C.Pradhan
HBCSE, TIFR, V.N. Purav Marg
Mankhurd, Mumbai

In this article (third in the series of articles) we present problems for a problem based learning course from the area of mechanics based on simple harmonic motion, rotational motion, gravitation and waves. We present the learning objectives in this area of basic physics and what each problem tries to achieve with its solution.

Introduction

In this article, third in the series of Problem Based Learning in Basic Physics, we present problems on simple harmonic motion, rotational motion, gravitation and waves. Methodology and philosophy of selecting these problems are already discussed. (Pradhan, 2009; Mody. 2011).

To review methodology in brief, we note here that this Problem Based Learning (PBL) starts after students have been introduced to formal structure of Physics. Ideally students would attempt only main problems. If they find it difficult, then depending upon their area of difficulty, right auxiliary problems have to be introduced by teacher who is expected to be a constructivist facilitator. Teacher may choose as per her/his requirement or may construct questions on the spot to guide students to right idea and method.

Problems

D: Simple Harmonic Motion

11. A cylindrical log of wood of height h and area of cross-section A floats in water. It is pressed and then released. Show that the log would execute SHM with a time period $T = 2\pi\sqrt{\frac{m}{A\rho g}}$ where m is the mass and ρ is the density of the liquid [NCERT EP XI].

Tasks involved in this problem are:

1. To understand the balance of forces when the log is in equilibrium.
2. To find the net force that would try to restore the log back to equilibrium and realise that it is proportional to displacement.

3. The constant of proportionality should be identified and its relation with time period would lead to the expression for time period.

Many such motions are equivalent and has simple behaviour, examples are liquid in a U-tube, magnets in magnetic field, marble in a hemispherical bowl , etc... . Treatment presented here helps understand such motion or any simple deviation that may be possible.

E : Rotational Motion

12. A carpet of mass M, made of inextensible material, is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part when its radius reduces to $R/2$ [JEE, 1990].

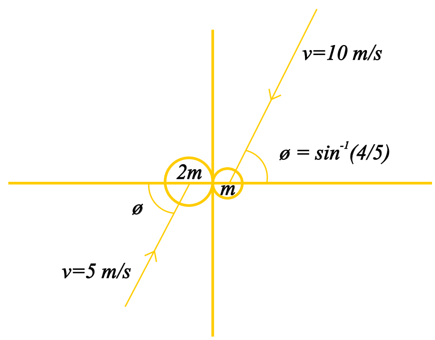
Tasks involved in this problem are:

1. To realise that mass of cylinder here changes as R^2 and keeps on changing as the carpet unrolls.
2. The unrolling part of carpet (cylindrical part) has linear as well as rotational motion.
3. The deciding factor here is the conservation of energy.

13. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at $t = 0$. After t_0 seconds, it acquires a purely rolling motion as shown. (i) Calculate the velocity of the centre of mass of the disc at t_0, and (ii) assuming the coefficient of friction to be μ, calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over time t much longer than t_0 [JEE, 1997].

Tasks involved in this problem are:

1. To identify force that causes sliding disc to rotate and roll.
2. This force causes slowing down (deceleration) of linear motion and torque due to which speeds up rotation of the disc thus to find condition for this.
3. To find the condition when rolling without slipping begins.
4. To identify contribution to kinetic energy initially and finally and hence change in kinetic energy.



14. Two smooth spheres made of identical material having masses m and $2m$ collide with an oblique impact as shown in figure above. The initial velocities of the masses are also shown. The impact force is along the line joining their centres. The coefficient of restitution is 5/9. Calculate the velocities of the masses after the impact and the loss in the kinetic energy [Basavraju and Ghosh].

Tasks involved in this problem are:

1. To understand that collision is inelastic but v-velocity is unaffected.

2. To apply conservation of momentum and find velocities in x- direction.
3. Hence, work out motion after the collision.

F : Gravitation

15. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day-to-day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain the variation of length of the day during the year? [NCERT, EP XI].

Tasks involved in this problem are:

1. To understand how planet moves under Kepler's law.
2. To apply Kepler's law and find angular speeds at apogee and perigee.
3. To understand mean rotation and length of the day in terms of angular speed.

16. A satellite is in an elliptic orbit around the earth with apehelion of $6R$ and perihelion of $2R$ where $R = 6400$ km is the radius of the earth. Find the eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius $6R$? [$G = 6.67\times10^{-11}$ SI units and $M = 6\times10^{24}$ kg] [NCERT EP XI].

Task involved in this problem is:

To apply conservation of angular momentum in central force (elliptic orbit) to find out how velocity changes as satellite moves in the orbit .

G : Waves

Learning objectives:

1. Becoming familiar with wave propagation and effect of distance ($I \propto 1/r^2$ law) and state of motion of observer or source (Doppler effect) on the observed parameters namely: amplitude, intensity, frequency, wavelength and phase.
2. Superposition of waves leading to beats, resonance and response of detector.
3. To understand mathematical structure dealing with above mentioned points.

17. The displacement of the medium in a sound wave is given by the equation $y_1 = A \cos (ax + bt)$, where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. *

(i) What are the wavelength and frequency of the incident wave?

(ii) Write the equation for reflected wave.

(iii) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

- It illustrates what happens when a wave is reflected, how do the incident and reflected waves interact and phase of the reflected wave
- It introduces students to waves and reflection on a surface.

Tasks involved in this problem are:

1. To apply principle of superposition.

2. To understand the phase of the reflected wave.
3. To find intensity using the rule that it is proportional to square of the amplitude.
    - No supporting problem. Students are to be guided to achieve the goal.
    - To recognise what the phase of the reflected wave would be they were asked to tell the difference between the waves reflected from the denser and rarer surface and how it gets incorporated mathematically into equation.

18. Two radio stations broadcast their programmes at the same amplitude A, and at slightly different frequencies $ù_1$ and $ù_2$ respectively, where $ù_2 - ù_1 = 10^3$ Hz. A detector receives the signal from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$.

(i) Find the time interval between two successive maxima of the intensity of the signal received by the detector.

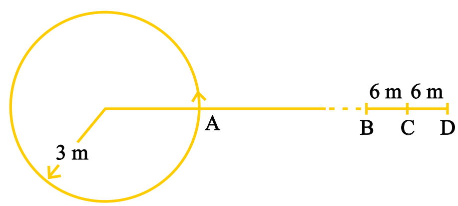
(ii) Find the time for which the detector remains idle in each cycle of intensity of the signal [JEE, 1993].

- This problem illustrates effect of beats on detection of waves.

Tasks involved in this problem are:

1. To represent two waves by proper equation.
2. To use the principle of superposition for finding resultant amplitude.
3. To find final intensity proportional to square of the amplitude.
4. To make decision as to when will detector be idle. For this, they may have to plot a graph of intensity v/s time.

19. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC = CD = 6 m. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and minimum frequencies recorded by the detector [JEE, 1990].



- This is an example based on Doppler effect. It is interesting as it makes use of symmetric movement of detector due to its simple harmonic motion.

Tasks involved in this problem are:

(1) To apply Doppler effect by incorporating relative motion of source and detector.

(2) To see and recognise the symmetry in motion between source and detector.

(3) To recognise the importance of the fact that source and detector are far away.

Solutions

D: Simple Harmonic Motion

11. Let the log be pressed and let the vertical displacement at the equilibrium position be x_0. At equilibrium *mg* = buoyant force = *Ax*$_0$ *ñg*.

When it is displaced further by *x*, the buoyant force is *A*(*x* + *x*$_0$)*ñg*.

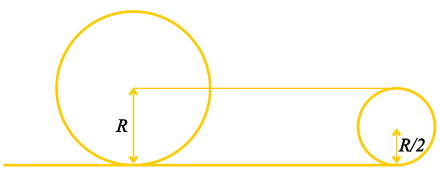
Therefore, the net restoring force = buoyant force - weight = *Axñg* (proportional to *x*)

Thus *mù* ^{2} = *Añg* and hence time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}}\ .$$

E : Rotational Motion

12. Mass of the cylindrical part of the carpet is proportional to R^2 and as centre of mass (axis) of the carpet gets lowered, its gravitational potential energy decreases and gets converted into kinetic energy of the rolling part.



P.E. lost $= MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{7}{8}MgR$

K. E. gained = K.E. of axis (CM) + rotational K.E. around axis

$= \frac{1}{2}\frac{M}{4}v_{CM}^2 + \frac{1}{2}I\omega^2$, where $v_{CM} = \omega\frac{R}{2}$

and $I = \frac{1}{2}\frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{1}{32}MR^2$

$= \frac{3}{16}Mv_{CM}^2$.

Comparing the two we get , $v_{CM} = \sqrt{\frac{14}{3}gR}$.

13. (i) Force of friction opposing motion of the disc is *f*$_r$ = *ì mg*

which causes acceleration of *a* = *ì g*

Torque acting on the disc due to friction is *ô* = *ì mgR*

and causes angular acceleration

á = *ì mgR*/*I* where *I* = ½*mR*^{2}

at time *t*$_o$, *v* = *Rù* ⇒ *v*$_o$ – *at*$_o$ = *R* (*ù* $_o$ + *át*$_o$)

v$_o$ = 3*ì mgt*$_o$ *t*$_o$ = *v*$_o$ / 3*ì mg*

ù = *at*$_o$ =2*v*$_o$/3*R* and *v* = *R ù* = 2*v*$_o$/3

(ii) W = *ΔKE* = ½*mv*^{2} + ½*Iù* ^{2} - ½*mv*$_o^2$

= (3/2) *mì* ^{2}*g*^{2}*t*(*t* – 2*t*$_o$)

At t = *t*$_o$: *W* = -(3/2) *mì* ^{2}*g*^{2}*t*$_o^2$ = -(1/6)*mv*$_o^2$:

work done by friction (decrease in KE).

14. denoting velocities for masses *2m* and *m* as *u*$_1$ and *u*$_2$ before and *v*$_1$ and *v*$_2$ after collision, we have

u_{2x} = -10cosϕ = -6m/s , u_{1y} = 5sinϕ = 4m/s

and u_{2y} = -10sinϕ = -8m/s

Since impact force is only along x- direction, y components of velocities remain unchanged.

$v_{1y} = u_{1y}$ = 4 m/s and $v_{2y} = u_{2y}$ = - 8 *m/s*.

Conservation of momentum in x-direction gives and coefficient of restitution

v_{1x} = -5/3 *m/s* and v_{2x} =10/3 *m/s*

and θ = *tan*$^{-1}$(*v*$_y$/*v*$_x$) = -12/5 indicates that

sphere 1 moves at 113° to x-axis and sphere 2 moves at 67° (in 4th quadrant) to x-axis after collision.

F : Gravitation

15. Angular momentum is conserved in motion under central force and as per Kepler's law area velocity is constant, i.e. r^2*ù* (at perigee) = r^2*ù* (at apogee) = constant

$$\therefore r_p^2 \omega_p = r_a^2 \omega_a .$$

If *a* is the semi-major axis of earth's orbit, then $r_p = a(1 - e)$ and $r_a = a(1 + e)$

$$\frac{\omega_p}{\omega_a} = \left(\frac{1+e}{1-e}\right)^2 = 1.0691 \text{ since } e = 0.0167.$$

If *ù* is the mean angular speed of the sun (1°) per day) then it corresponds to mean solar day.

$$\therefore \left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691$$

λ

$$\therefore \left(\frac{\omega_p}{\omega}\right) = \left(\frac{\omega}{\omega_a}\right) = 1.034$$

ù $_p$ = *1.034°* per day and *ù* $_a$ = *0.967°* per day.

If we consider 361° rotation of earth for 1 day then at perigee day is 8.1" longer and at apogee it is 7.9" smaller.

This does not explain the actual variation of length of the day during the year.

16. $r_a = a(1 + e) = 6R$ and $r_p = a(1 - e) = 2R$
$\Rightarrow e = ½$

Angular momentum at perigee = Angular momentum at apogee

$mv_p r_p = mv_a r_a \qquad v_a = (1/3) v_p$

Energy at perigee = Energy at apogee

$$\frac{1}{2}mv_p^2 - \frac{GM}{r_p} = \frac{1}{2}mv_a^2 - \frac{GM}{r_a}$$

$$\therefore v_p \frac{2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]^{½}}{\left[1 - \left(\frac{v_a}{v_p}\right)\right]^2} = \left[\frac{\frac{2GM}{R}\left[\frac{1}{2} - \frac{1}{6}\right]}{\left[1 - \frac{1}{9}\right]}\right]^{½}$$

$$\sqrt{\frac{3GM}{4R}} = 6.85 \ km/s$$

$v_a = 2.28 \ km/s$

.

For $r = 6R, \quad V_c = \sqrt{\frac{GM}{6R}} = 3.23 \ km/s$

Thus transfer of orbit requires *Äv = 0.95 km/s,* can be achieved by firing rockets from the satellite.

G : Waves

17. Stationary Waves

Incident wave : $y_1 = A \cos(ax + bt)$

(i) comparing this with $y = a \cos(2\pi nt - 2\pi x/$

we get, frequency $n = b/2p$ and wavelength $l = 2\pi/a$

(ii) Reflected wave: $y_2 = 0.8 A \cos(ax - bt + \pi)$

Intensity of the reflected wave is 0.64 times and hence amplitude is 0.8 times that of the incident wave. Sign change due to reflection and additional phase of π due to reflection on denser surface.

(iii) Particle speeds are *dy/dt* and will be maximum at antinodes and minimum at nodes.

18. Beats

$w_2 - w_1 = 10^3 Hz$ Beat frequency

(i) $T = (w_2 - w_1)^{-1} = 1$ ms Beat period.

(ii) $y_1 = A \sin 2\pi w_1 t$ and $y_2 = A \sin 2\pi w_2 t$.

Resultant wave $y = y_1 + y_2 = R \cos 2\pi[(w_1 + w_2)/2]\, t$

where $R = 2A \sin \pi(w_2 - w_1)\, t$.

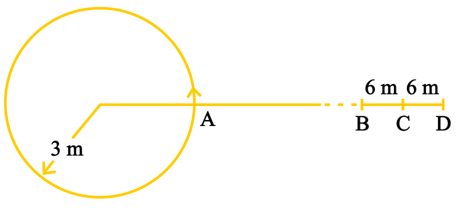
For detector response $I \geq 2A^2$ or $R \geq \pm\sqrt{2}\, A$.

This is when $\sin \pi(w_2 - w_1)\, t = \pm \frac{1}{\sqrt{2}}$

$\pi(w_2 - w_1)\, t = \pi/4, 3\pi/4, 5\pi/4, \ldots$

Therefore, detector remains idle for $\ddot{A}t = T/2$
$= 1/2(w_2 - w_1) = 0.5$ ms

19. Doppler Effect



Detector is far away from the source and hence radius of the signal becomes insignificant.

Both source frequency of circular motion and detector frequency of SHM are identical.

Source is at A (no longitudinal motion) when detector is at B (at momentary rest) and hence no Doppler shift.

When Detector is at C moving towards D, source is also moving away, hence minimum frequency get recorded: $n_{\min} = n_o \frac{(v - u_{detector})}{(v + u_{source})}$

When Detector is at C moving towards B, source is also moving towards hence maximum frequency get recorded:

$$n_{\max} = n_o \frac{(v + u_{detector})}{(v - u_{source})}$$

* Source unknown

References

BASAVRAJU, G. and D. GHOSH. 1988. *Mechanics and Thermodynamics.* Tata McGraw-Hill, New Delhi.

JEE — Joint Entrance Examination for admission to IIT.

MODY, A. K. and H.C. PRADHAN. 2011. *Problem Based Learning in Basic Physics - I*, School Science, **49** (3).

PRADHAN, H.C. and A.K. MODY. 2009. *Constructivism Applied to Physics Teaching for Capacity Building of Undergraduate Students*, University News, **47** (21), 4-10.

NCERT, 2006. Physics Textbook for class XI part-I, New Delhi.

NCERT, 2006. Physics Exemplar Problems for Class XI, New Delhi.